4th Latin American SCAT Workshop Facultad de Ciencias Físicas y Matemáticas Universidad de Chile

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In the past fifteen years, the framework and ideas from dynamical systems theory have been applied to a variety of transport and mixing problems in oceanic flows. The motivation for this approach comes directly from advances in observational capabilities in oceanography (e.g., drifter deployments, remote sensing capabilities, satellite imagery, etc.) which reveal space-time structures that are highly suggestive of the structures one visualizes in the global, geometrical study of dynamical systems theory.

In this course, we motivate this approach by showing the relationship between fluid transport in two-dimensional time-periodic incompressible flows and the geometrical structures that exist for two-dimensional area-preserving maps, such as hyperbolic periodic orbits, their stable and unstable manifolds and KAM (Kolmogorov-Arnold-Moser) tori. This serves to set the stage for the attempt to "transfer" this approach to more realistic flows modeling the ocean. However, in order to accomplish this several difficulties must be overcome.

The first difficulty that confronts any attempt to carry out a dynamical systems approach to transport requires us to obtain the appropriate "dynamical system", which is the velocity field





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describing the fluid flow. In general, adequate model velocity fields are obtained by numerical solution of appropriate partial differential equations describing the dynamical evolution of the velocity field. Numerical solution of the partial differential equations can only be done for a finite time interval, and since the ocean is generally not time-periodic, this leads to a new type of dynamical system: a finitetime, aperiodically time-dependent velocity field defined as a data set on a space-time grid.

The global, geometrical analysis of transport in such dynamical systems requires both new concepts and new analytical and computational tools, as well as the necessity to discard some of the standard ideas and results from dynamical systems theory.

This course will describe these new concepts and analytical tools first using simple dynamical systems where quantities can be computed exactly. We then discuss their computational implications and implementation in the context of three specific geophysical flows: a general circulation model of the Mediterranean Sea, a turbulent wind-driven double-gyre in the quasigeostrophic approximation, and a representation of the velocity field of Monterey Bay obtained from high frequency radar observations. In the few lectures we will go into more detail about the numerical methods, motivated by the examples.





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